Volatility Smiles

Explain how put-call parity indicates that the implied volatility used to price call options is the same used to price put options.

■ Put-call parity indicates that the deviation between market prices and Black-Scholes-Merton prices will be equivalent for calls and puts. Hence, implied volatility will be the same for calls and puts.

1. Put-call parity:

$$p + S_0 e^{-qT} = c + Ke^{-rT}$$

2. It is based on a relatively simple on no-arbitrage argument. It doesn't require any assumption about the probability distribution of the asset price in the future. therefore

$$p_{BS} + S_0 e^{-qT} = c_{BS} + Ke^{-rT}$$

$$p_{mkt} + S_0 e^{-qT} = c_{mkt} + Ke^{-rT}$$

Subtracting these two equations, we get

$$p_{BS} - p_{mkt} = c_{BS} - c_{mkt} \quad (*)$$

3. Suppose that the implied volatility of the put option is 22%. This means that $p_{BS} = p_{mkt}$ when a volatility of 22% is used in the Black-Scholes model. From equation *, it follows that $c_{BS} = c_{mkt}$ when this volatility is used. The implied volatility of the call is , therefore, also 22%

Q1. An analyst observes that the market price of a call option is USD 5 higher than the theoretical price dictated by an option pricing model. Assuming the same strike price and time to expiration, what should be the relationship between the market price of a put option and its theoretical price dictated by the option pricing model?

  a. The market price will be higher than the theoretical price by USD 5.

  b. The market price will be higher than the theoretical price by an amount lower than USD 5.

  c. The market price will be lower than the theoretical price by USD 5.

 d. The market price will be lower than the theoretical price by an amount lower than USD 5.

Ans.: *A*

Define volatility smile and volatility skew.

Relate the shape of the volatility smile (or skew) to the shape of the implied distribution of the underlying asset price and to the pricing of options on the underlying asset.

Explain why foreign exchange rates are not necessarily lognormally distributed and the implications this can have on option prices and implied volatility.

■   Volatility smile

The volatility pattern used by traders to price currency options generates implied volatilities for away-from-the-money options are greater than at-the-money options.
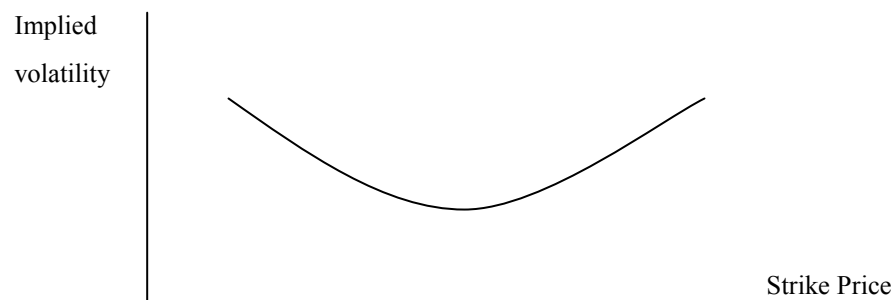
■   The implied distribution has heavier tails and is more peaked than the lognormal distribution. Both small and large movements in the exchange rate are more likely than with lognormal distribution. Intermediate movements are less likely.
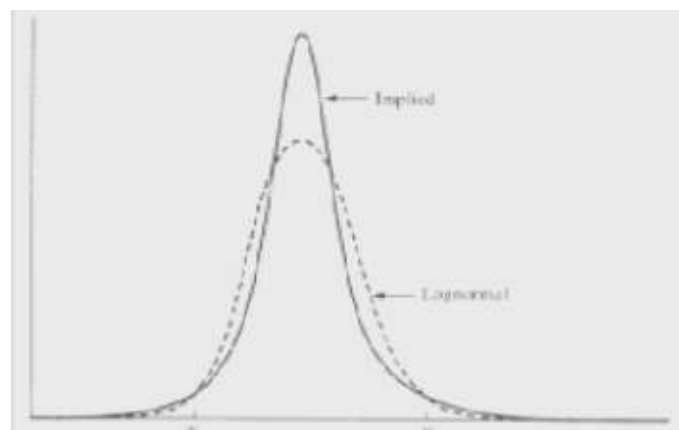

■   The two conditions for an asset price to have a lognormal distribution
  ◆ The volatility of the asset is constant
 ◆ The price of the asset changes smoothly with no jumps

■ The tendency for exchange rate changes to be more extreme is a function of the fact that exchange rate volatility is not constant and frequently jumps from one level to another. However, these two effects tend to be mitigated for long-date options, which tend to exhibit less of a volatility smile pattern than shorter-dated options.

Discuss the volatility smile for equity options and give possible explanations for its shape.

■ Volatility smile in equity options ("smirk" or "skew")
  ◆ Shows a higher implied volatility for low strike price options (in-the-money calls and out-of-the-money puts) than for high strike price options (in-the-money puts and out-of-the-money calls).

  ◆ There is essentially an inverse relationship between implied volatility and strike price of equity options.
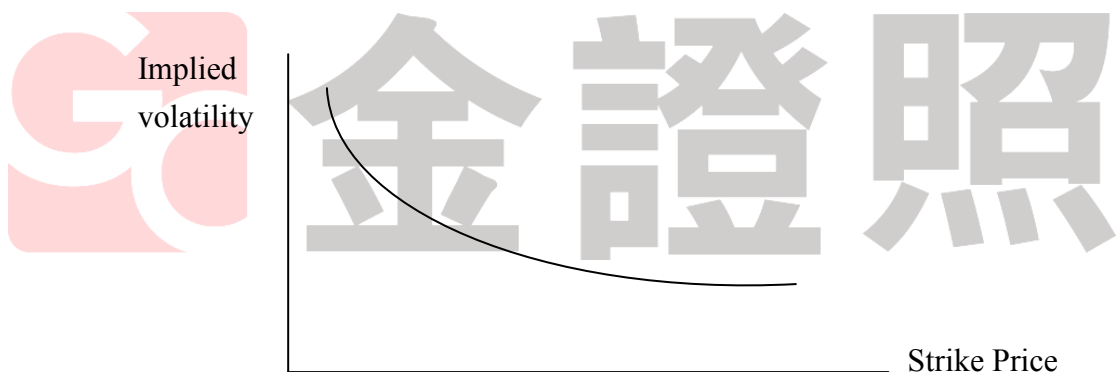
■ The implied distribution has a heavier left tail and a less heavy right tail than lognormal distribution.
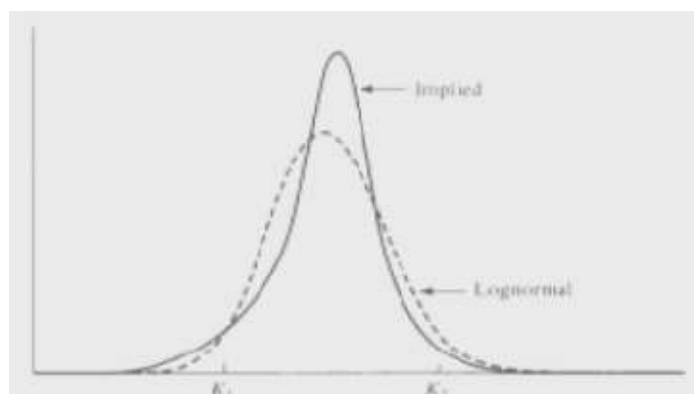
- ■ Two reason:
  - ◆ Leverage
    - Equity value ↓, leverage ↑ , Risk ↑ ,volatility of equity ↑
    - Equity value ↑, leverage ↓ , Risk ↓ ,volatility of equity ↓
    - we can expect the volatility of equity to be a decreasing function of price.

  - ◆ Crashophobia
    - Market participants are simply afraid of another market crash, so they place a premium on the probability of stock prices falling precipitously. Put prices will exhibit a higher premium at the lower strike price – deep- out-of-the-money puts provide protection against a substantial drop in equity prices; deep in-the-money-calls allow for more payoff if equity prices rise.

Q2.Which one of the following statements is incorrect regarding the properties of volatility smiles?
- a. One possible reason for the smile in equity options is that people are concerned about stock market crashes.
- b. Referring to the volatility smile of foreign currency options, implied volatility of at-the-money options is relatively low and it becomes higher for the deep in-the-money options and deep out-of-the-money options.
- c. A volatility smile in foreign currency options exists because traders think that the lognormal distribution underestimates the likelihood of an extreme exchange rate movement.
- d. The implied volatility of equity options increases as the strike price increases.

Ans.: *D*

Q3.With all other things being equal, a risk monitoring system that assumes constant volatility for equity returns will understate the implied volatility for which of the following positions by the largest amount:
- a. Short position in an at-the-money call
- b. Long position in an at-the-money call
- c. Short position in a deep in-the-money call
- d. Long position in a deep in-the-money call

Ans.: *D*